WAVES   Introduction, Classification & Wave Equation
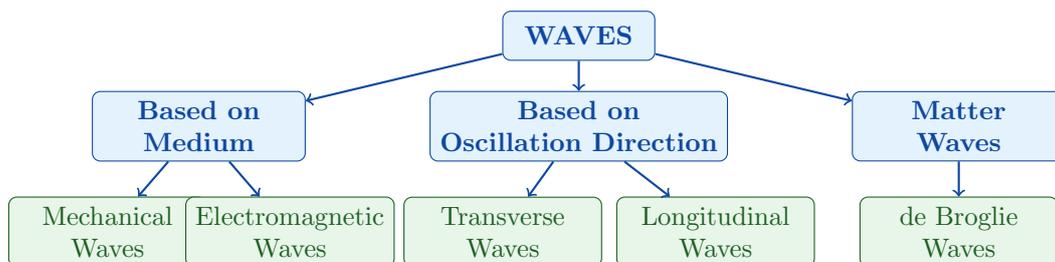
i) Introduction

Definition: A **wave** is a disturbance that transfers **energy** from one point to another *without* the actual transfer of matter (particles) of the medium. The particles of the medium only oscillate about their mean positions.

Key Points:
- It is **energy** that travels, not matter.
- Every wave requires a **source of disturbance**, a **medium** (except EM waves), and a **mechanism** for energy transfer.
- Wave speed $\neq$ particle speed. Wave speed depends on the **medium**; particle speed depends on amplitude & frequency.

ii) Classification of Waves

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                        WAVES
         ┌───────────────┼───────────────┐
    Based on        Based on          Matter
    Medium       Oscillation Direction  Waves
    ┌────┴────┐      ┌────┴────┐          │
Mechanical Electromagnetic Transverse Longitudinal  de Broglie
  Waves      Waves       Waves       Waves         Waves
```

Mechanical Waves

Waves that **require a material medium** to propagate.

Conditions needed:
- Inertia of the medium (mass)
- Elasticity of the medium
- Minimum friction / damping

Examples: Sound waves, water waves, waves on a string, seismic waves.

Electromagnetic Waves

Waves that **do not require any medium** (can travel through vacuum).

- Speed in vacuum: $c = 3 \times 10^8$ m/s
- Always **transverse** in nature
- Produced by accelerating charges

Examples: Light, radio waves, X-rays, microwaves, γ-rays.

Transverse Waves

Particles oscillate **perpendicular** ($\perp$) to the direction of wave propagation.

- Show **crests** and **troughs**
- Travel in solids and on liquid surfaces
- **Cannot** travel through gases or liquid bulk
- All EM waves are transverse

Examples: Light waves, waves on a stretched string.

Longitudinal Waves

Particles oscillate **parallel** ($\parallel$) to the direction of wave propagation.

- Show **compressions** and **rarefactions**
- Can travel in **all three states** of matter
- Require bulk (volume) elasticity

Examples: Sound waves in air, waves in a spring (slinky).

iii)   Representation of a Wave

General Equation of a Progressive (Travelling) Wave:

$$y(x,\,t) \;=\; A\sin(\omega t - kx + \phi)$$

Symbol	Meaning	Formula / Unit
y	Displacement of particle	metres (m)
A	Amplitude — max displacement	metres (m)
ω	Angular frequency	$\omega = 2\pi f = \dfrac{2\pi}{T}$ rad/s
k	Wave number (propagation const.)	$k = \dfrac{2\pi}{\lambda}$ rad/m
ϕ	Initial phase	radians
$(\omega t - kx + \phi)$	Instantaneous phase	radians

Important Relations

$$v = f\lambda = \frac{\omega}{k}, \qquad T = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$v_{\text{particle}} = \frac{\partial y}{\partial t} = A\omega \cos(\omega t - kx + \phi)$$

$$(v_{\text{particle}})_{\max} = A\omega$$

Direction of Wave Travel
- $y = A\sin(\omega t - kx) \Rightarrow$ travels in $+\mathbf{x}$ direction
- $y = A\sin(\omega t + kx) \Rightarrow$ travels in $-\mathbf{x}$ direction
- $v_p = -\,v_{\text{wave}} \cdot \dfrac{\partial y}{\partial x}$

$$v = f\lambda \quad | \quad \omega = 2\pi f \quad | \quad k = \frac{2\pi}{\lambda} \quad | \quad (v_p)_{\max} = A\omega \quad | \quad \Delta\phi = \frac{2\pi}{\lambda}\,\Delta x = \omega\,\Delta t$$